Watershed Sciences 6900/6840
FLUVIAL HYDRAULICS & ECOHYDRAULICS

VELOCITY DISTRIBUTIONS,
FLOW RESISTANCE &
DERIVATION OF CHEZY

Joe Wheaton
TODAY’S PLAN

VELOCITY DISTRIBUTIONS

I. Velocity Profile in Laminar Flows
II. Velocity Profile in Turbulent Flows
   I. Prandtl von Karman Velocity Profile
   II. Velocity Defect Law
   III. Power Law Profiles
III. Observed Velocity Distributions
Previous chapters have discussed the velocity of individual fluid elements (point velocities), denoted as $u_x$, $u_y$, $u_z$, and the average velocity through a stream cross section, denoted as $U$. The main objective of the present chapter is to explore the connection between point velocities and cross-section average velocity by developing physically sound quantitative descriptions of the distribution of velocity in cross sections.

However, there has been little research on the distribution of velocities in entire cross section, so most of the discussion here will be devoted to velocity profiles:

Most of discussion limited to vertical velocity profiles...

The **velocity profile** is the relation between downstream-directed velocity $u(y)$ and normal distance above the bottom, $y$.\(^1\)
WHY DO WE CARE?

• Basis for formulating expressions of flow resistance

\[ F_D = F_R \]
WHAT IS A VELOCITY PROFILE?

• A mathematical function for velocity as a function of depth (or height above bed): \( u(y) = ? \)

\[
U_w = \frac{1}{Y_w} \cdot \int_0^{Y_w} u(y) \cdot dy
\]

\[
U = \frac{1}{A} \cdot \int_0^A U_w(w) \cdot dA(w)
\]

• \( U_w \) - Local average ‘vertical’ velocity
• \( U \) - Average cross section velocity
• \( Y_w \) - Local Depth
• \( A \) - Cross Sectional Area
• \( w \) - Water surface width
HOW DO I MEASURE A VELOCITY PROFILE?

- You take a bunch of points...
- Say every few centimeters?
Depth-Slope Product: Boundary Shear Stress:

\[ \tau_0 = \gamma \cdot Y_w \cdot \sin \theta_S \]

Linear shear stress relationship for both laminar & turbulent flow:

Figure 5.2 (a) The linear relation between shear stress, \( \tau \), and distance above the bottom, \( y \), given by equation 5.6. This relation applies to both laminar and turbulent flow states. (b) Shear stress distribution in a turbulent flow. The shaded area schematically represents the portion of total shear stress that is due to molecular viscosity. Total shear stress is the sum of that due to molecular viscosity and that due to eddy viscosity.
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LAMINAR FLOW - PROFILE

\[ u(y) = \frac{\gamma}{\mu} \cdot \left( Y_w \cdot y - \frac{y^2}{2} \right) \cdot \sin \theta_s \]

To visualize this distribution, we can first use equation 5.11 to calculate the velocity at the surface, \( u(Y_w) \):

\[ u(Y_w) = \frac{\gamma}{\mu} \cdot \left( \frac{Y_w^2}{2} \right) \cdot \sin \theta_s. \]  

(5.12)

Then we can plot the dimensionless relative velocity \( u(y)/u(Y_w) \) versus relative distance above the bottom, \( y/Y_w \), in figure 5.3, where from equation 5.11 and 5.12,

\[ \frac{u(y)}{u(Y_w)} = 2 \cdot \frac{y}{Y_w} - \left( \frac{y}{Y_w} \right)^2. \]  

(5.13)

Figure 5.3 Relative velocity \( u(y)/u(Y_w) \) as a function of relative distance above the bottom, \( y/Y_w \), for laminar open-channel flows (equation 5.13).
WHAT’S HE TELLING US?

- Average vertical velocity

5.2.2 Average “Vertical” Velocity

The average local “vertical” velocity of a wide laminar flow, $U_w$, is given by substituting equation 5.11 into 5.2 and integrating; evaluating that expression leads to

$$U_w = \left( \frac{\gamma}{3 \cdot \mu} \right) \cdot Y_w^2 \cdot \sin \theta_s. \tag{5.14}$$

Recall from section 3.4.2 that laminar flow only occurs when the Reynolds number, $Re$, is less than 500, where

$$Re \equiv \frac{U_w \cdot Y_w}{v}. \tag{5.15}$$

If we substitute 5.14 into 5.15 and recall that $v \equiv \mu / \rho$ and $\gamma = \rho \cdot g$, we arrive at

$$Re = \frac{g \cdot Y_w^3 \cdot \sin \theta_s}{3 \cdot v^2}. \tag{5.16}$$

and if $Re = 500$, the limiting value for laminar flow, we have

$$Y_w = \left( \frac{1500 \cdot v^2}{g \cdot \sin \theta_s} \right)^{1/3}. \tag{5.17}$$

We can use equation 5.17 to find the maximum depth for which a flow will be laminar at a specified slope; this relation is shown in figure 5.4. Note that even for surfaces with very low slopes (e.g., parking lots), this depth is in the centimeter range; for hillslopes, for which typically $\sin \theta_s > 0.01$, the maximum depth is in the millimeter range.

Figure 5.4 Maximum depth at which laminar flow occurs as a function of slope (equation 5.17). Kinematic viscosity is assuming a water temperature of 10°C.
TODAY’S PLAN

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SO IN TURBULENT FLOWS....

- It’s most commonly treated with the ‘Law of the Wall’ (i.e. Prandtl von Karman)
- For practical purposes... it is very difficult to measure the viscous sublayer in natural streams
WHAT DOES IT MEAN?

- In log-log space, a straight line means?
- The profile is logarithmic...
TODAY’S PLAN

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DERIVATION OF Prandtl-von Kármán (P-vK)

• Starting with equations 3.40a & 5.6 (in sections you will have read but we did not cover…

• Bunch of math magic…

\[ u(y) = \left(\frac{1}{\kappa}\right) \cdot (g \cdot Y_w \cdot \sin \theta_s)^{\frac{1}{2}} \cdot \ln \left(\frac{y}{y_0}\right) \]

• Where you know everything… except \( \kappa \)

• In practice \( \kappa = 0.4 \)
• Can range from 0.2 to 0.4
SHEAR VELOCITY (FRICTION VELOCITY)

\[ u(y) = \left( \frac{1}{k} \right) \cdot (g \cdot Y_w \cdot \sin \theta_s) \frac{1}{2} \cdot \ln \left( \frac{y}{y_0} \right) \]

5.3.1.3 Shear Velocity (Friction Velocity)

The quantity \((g \cdot Y_w \cdot \sin \theta_s)^{1/2}\) in equation 5.21 has the dimensions of a velocity. This quantity is called the shear velocity, or friction velocity, designated \(u_s\):

\[ u_s = (g \cdot Y_w \cdot \sin \theta_s)^{1/2}. \] (5.24)

The shear velocity is a measure of the intensity of turbulent velocity fluctuations. To see this, recall from equation 3.32 that the shear stress at a height \(y\) above the bed in a turbulent flow, \(\tau(y)\), is related to the average turbulent velocity fluctuations as

\[ \tau(y) = -\rho \cdot \bar{u}_x'(y) \cdot \bar{u}_y'(y), \] (5.25)

where \(\bar{u}_x'(y)\) and \(\bar{u}_y'(y)\) are the average fluctuations in the \(x\)- and \(y\)-directions, respectively. We also saw from equation 3.31 that the magnitudes of these fluctuations are proportional, so we can write

\[ \tau(y) = -k_{yy} \cdot \rho \cdot [\bar{u}_x'(y)]^2, \] (5.26)

where \(k_{yy}\) is the proportionality constant. Now, noting equation 5.7, we see that

\[ u_* = \left( \frac{\tau_0}{\rho} \right)^{1/2}, \] (5.27a)

and

\[ \tau_0 = \rho \cdot u_s^2. \] (5.27b)

Comparing equation 5.27 with 5.26, we see that in turbulent flows \(u_s\) and \(\tau_0\) are alternate ways of expressing both the intensity of turbulence and the boundary shear stress. Shear velocity \(u_s\) expresses these physical quantities in kinematic (velocity) terms, whereas \(\tau_0\) expresses them in dynamic (force) terms. Also note that \(u_s\) can be thought of as a characteristic near-bed velocity in a turbulent flow.
RELATIVE VELOCITY PROFILE
(DIMENSIONLESS)

Figure 5.5 Relative velocity $u(y)/u(Y_w)$ as a function of relative distance above the bottom, $y/Y_w$, as given by the Prandtl-von Kármán universal velocity distribution (equation 5.21) for turbulent open-channel flow with a depth $Y_w = 1$ m and a slope $\sin \theta = 0.001$. 
ROUGNNESS REYNOLDS NUMBER

Recall:

\[ Re \equiv \left( \frac{\rho \cdot Y \cdot U}{\mu} \right) = \left( \frac{Y \cdot U}{\nu} \right) \]

Let thickness of the sublayer \( y_r \) be the characteristic length... Then, the boundary Reynolds number (or roughness Re) is:

\[ Re_b \equiv \left( \frac{y_r \cdot u_*}{\nu} \right) \]

Experiments have shown that \( Re_b \):

- Smooth: \( Re_b > 5 \)
- Transitional: \( 5 > Re_b > 70 \)
- Rough: \( Re_b > 70 \)
5.3.1.8 The P-vK Law: Summary

To summarize the discussions of sections 5.3.1.2–5.3.1.7, we use equations 5.24 and 5.32 to write the P-vK law in the forms that we will usually apply it:

**Smooth flows, \( Re_b \leq 5 \):**

\[
\frac{u(y)}{u_*} = 2.50 \cdot \ln \left( \frac{9 \cdot u_* \cdot y}{v} \right); \quad (5.34a)
\]

**Rough flows, \( Re_b > 5 \):**

\[
\frac{u(y)}{u_*} = 2.50 \cdot \ln \left( \frac{30 \cdot y}{y_r} \right). \quad (5.34b)
\]

Note that these are mathematically equivalent to **Smooth flows, \( Re_b \leq 5 \):**

\[
\frac{u(y)}{u_*} = 2.50 \cdot \ln \left( \frac{u_* \cdot y}{v} \right) + 5.49 = 5.76 \cdot \log \left( \frac{u_* \cdot y}{v} \right) + 5.49; \quad (5.34c)
\]

**Rough flows, \( Re_b > 5 \):**

\[
\frac{u(y)}{u_*} = 2.50 \cdot \ln \left( \frac{y}{y_r} \right) + 8.50 = 5.76 \cdot \log \left( \frac{y}{y_r} \right) + 8.50; \quad (5.34d)
\]

and the P-vK law may be written in any of these forms.
Starting with a definition of the average vertical velocity, derived by integration P-vK law over its range of validity (i.e. above the top of the buffer zone):

\[ U_w = \frac{1}{Y_w - Y_b} \cdot \int_{Y_b}^{Y_w} 2.50 \cdot u_* \cdot \ln \left( \frac{y}{y_0} \right) \cdot dy \]

Using the facts that

\[ \ln \left( \frac{y}{y_0} \right) = \ln(y) - \ln(y_0) \]
• 0.368 ≈ 0.4
• The six-tenths rule...
• If velocity profile follows P-vK (i.e. logarithmic) then I can just take one measurement!

Figure 5.10 P-vK velocity profile for a turbulent flow with $Y_w = 1$ m, showing velocity measurement by current meter at six-tenths of the depth measured from the surface. According to the P-vK law, the actual velocity $u(y)$ equals the average velocity $U_w$ at $y/Y_w = 0.368$.... This is the basis for the “six-tenths-depth rule” for measuring local average “vertical” velocity.

the basis for the six-tenths-depth rule used by the U.S. Geological Survey and others for discharge measurement:

If the P-vK law applies, the average velocity $U_w$ at a point in a cross section is found by measuring the velocity six-tenths of the total depth downward from the surface, or four-tenths ($≈ 0.368$) of the depth above the bottom (figure 5.10).
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From Chanson (2004)
A BORING RECTANGULAR FLUME...

Figure 5.19 Measured and simulated velocities and central velocity profiles in two flow rectangular flumes with low width/depth ratios, showing suppression of locus of maximum flow. (a) Vertical velocity profile in center. (b) Isolines show cross-section velocities in cm/s. Reproduced with permission of Elsevier.
SOMETHING MORE INTERESTING...

Figure 5.20 Isovels (cm/s) in a meander bend of the River Klarälven, Sweden, showing typical pattern of highest velocities in deepest portion of the cross section leading to helicoidal flow as shown in figure 5.21a. Note vertical exaggeration. From Sundborg (1956); reproduced with permission of Blackwell.

(a)

Point bar deposition

(b)

Figure 5.21 (a) Diagram of a meander bend (vertically exaggerated), showing typical asymmetry, helicoidal flow, point-bar deposition on inside of bend, and superelevation Δz. (b) Diagrammatic plan view of successive meander bends showing trace of thread of maximum velocity.
HOW GOOD IS Pv-K?

Error propagation for velocity and shear stress prediction using 2D models for environmental management

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KEYWORDS
Shear stress; 2D models; river restoration; hydraulics

Summary Resource managers, scientists, government regulators, and stakeholders are considering sophisticated numerical models for managing complex environmental problems. In this study, observations from a river restoration experiment involving gravel augmentation and spawning habitat enhancement were used to assess sources and magnitudes of error in depth, velocity, and shear velocity predictions made at the 1 cm scale with a commercial two-dimensional (depth-averaged) model. Error in 2D model depth prediction averaged 2%. This error was attributable to topographic survey resolution, which at 1 cm per 1.14 m², was inadequate to resolve small hillocks and depressions influencing point measurements. Error in 2D model velocity prediction averaged 2%. More than half of this error was attributable to depth prediction error. Despite depth and velocity error, 56% of 2D model predictions of shear velocity were within the 95% confidence limit of the best field-based estimation method. Ninety percent of the error in shear velocity prediction was explained by velocity prediction error. Multiple field-based estimates of shear velocity differed by up to 140%, so the lower error for the 2D model predictions augments such models are at least as accurate as field measurement. 2D models enable detailed, spatially distributed estimates compared to the small number measurable in a field campaign of comparable cost. They also can be used for design evaluation. Although numerical models are limited to channel types adhering to model assumptions and yield predictions only accurate to ±20–30%, they can provide a useful tool for river rehabilitation design and assessment, including spatially diverse habitat heterogeneity as well as for pre- and post-project appraisal.

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SOME EMPIRICAL DATA...
TODAY’S PLAN

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• I’ll post this tonight... (Chapter 6 – Uniform Flow & Flow Resistance)
• This week’s lab... more time to work through Chapter 6
• Next week, get back to something more fun in lab... flume experiment
TODAY’S PLAN

FUNDAMENTALS OF FLUVIAL HYDRAULICS

I. Terminology Review
II. Reminder of 1D – Mannings & Resistance

III. Conservation Laws -> Motion
   I. Conservation of Mass
   II. Conservation of Momentum

III. Conservation of Energy

IV. 1-D Step-Backwater Equations
V. 2D-3D – Navier Stokes & Others
VI. Summary
VELOCITY DISTRIBUTIONS

Figure 5.8. Main features of the structure of turbulent boundary layers. Modified from Bridge (2003).
TURBULENT BURSTS & SWEEPS & $U(y)$
DINGMAN’S ASSERTION...

“The central problem of open-channel-flow hydraulics can be stated as follows: Given a channel reach with a specified geometry, material, and slope, what are the relationships among flow depth, average velocity, width and discharge?”

Knowns:  
\[ z(x, y), S_0, d_0 \]

Unknowns:  
\[ Y, U, W, Q \]

In this chapter, we consider cross-section-averaged or reach-averaged conditions rather than local “vertically” averaged velocities \( (U_w) \) and local depths \( (Y_w) \), and will designate these larger scale averages as \( U \) and \( Y \), respectively. Figure 6.1 shows the spatial scales typically associated with these terms. Since our analytical reasoning will be based on the assumption of prismatic channels, there is no distinction between cross-section averaging and reach averaging. We will often invoke the wide...
TODAY’S PLAN

UNIFORM FLOW & FLOW RESISTANCE

I. Boundary Characteristics (for the chapter anyway)
II. Uniform Flow in Open Channels
III. The Chézy Equation
IV. Reach Resistance
    I. In Uniform Flow
    II. In Natural Channels
V. Mannings Equation
VI. Applying Resistance Equations

From Chanson (2004)
TODAY’S PLAN

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From Chanson (2004)
Let thickness of the sublayer $y_r$ be the characteristic length... Then, the boundary Reynolds number (or roughness Re) is:

Experiments have shown that $Re_b$:

- Smooth: $Re_b > 5$
- Transitional: $5 > Re_b > 70$
- Rough: $Re_b > 70$

The characteristic length $y_r$ is our **roughness height** (sometimes referred to as $k_0$).
“In natural alluvial channels, the bed material usually consists of sediment grains with a range of diameters. For a particular reach the characteristic height $y_r$ is usually determined as:”

$$y_r = k_r \cdot d_p$$  \hspace{1cm} \text{Eq. 6.2}$

- $d_p$ Diameter of particles larger than $p$ percent of the particles on the boundary surface
- $k_r$ A multiplier $\geq 1$
- We’ll often assume $k_r = 1$ & $p = 84$ so that $y_r = d_{84}$

From Dingman (2008) – Chapter 2 & 6
TODAY’S PLAN

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From Chanson (2004)
DEFINITION OF UNIFORM FLOW

• **Uniform Flow:** if the *element velocity* at any instant is constant along a streamline the flow is uniform (i.e. convective acceleration $du/dx = 0$).

• **Steady Flow:** if the *element velocity* $u$ at any given point on a streamline does not change with time, the flow is steady (i.e. local acceleration $du/dt = 0$). From Dingman (2008) – Chapter 4 & 6
As noted by Chow (1959, p. 89), unsteady uniform flow is virtually impossible of occurrence. Thus, henceforth, “uniform flow” implies “steady uniform flow.” Note, however, that a nonuniform flow may be steady or unsteady.

We will usually assume that the discharge, $Q$, in a reach is constant in space and time, where

$$Q = W \cdot Y \cdot U,$$

(6.3)

$W$ is the water-surface width, and $Y$ is average depth.

In uniform flow with spatially constant $Q$, it must also be true that depth and width are constant, so “uniform flow” implies $dY/dX = 0$ and $dW/dX = 0$. And, since the depth does not change, “uniform flow” implies that the water-surface slope is identical to the channel slope. Thus, it must also be true that for strictly uniform flow, cross-section shape is constant through a reach (i.e., the channel is prismatic).

$$F_D = F_R$$
RECALL NAVIER STOKES

- Conservation of Momentum... in 2D:
  \[ m \cdot a = \sum \vec{F} \]
  \[
  \rho \left( \frac{\partial U_x}{\partial t} + U_x \frac{\partial U_x}{\partial x} + U_y \frac{\partial U_x}{\partial y} \right) = -\rho g \frac{\partial z}{\partial x} - \frac{\partial P}{\partial x} + \vec{F}_{visc_x} \\
  \rho \left( \frac{\partial U_y}{\partial t} + U_x \frac{\partial U_y}{\partial x} + U_y \frac{\partial U_y}{\partial y} \right) = -\rho g \frac{\partial z}{\partial y} - \frac{\partial P}{\partial y} + \vec{F}_{visc_y}
  \]

- Simplified to 1D:
  \[
  \rho \left( \frac{\partial U_x}{\partial t} + U_x \frac{\partial U_x}{\partial x} \right) = -\rho g \frac{\partial z}{\partial x} - \frac{\partial P}{\partial x} + \vec{F}_{visc_x}
  \]

- So if flow is uniform, i.e. \( \frac{du}{dx} = 0 \), what happens?
WHOAA... IT SORTA WORKS...

\[
\rho \left( \frac{\partial U_x}{\partial t} + U_x \frac{\partial U_x}{\partial x} \right) = -\rho g \frac{\partial z}{\partial x} - \frac{\partial P}{\partial x} + \vec{F}_{visc_x}
\]

- So if flow is uniform, i.e. \( \frac{du}{dx} = 0 \), what happens?

\[
0 = -\rho g \frac{\partial z}{\partial x} - \frac{\partial P}{\partial x} + \vec{F}_{visc_x}
\]

\[
0 = -\vec{F}_{BODY} + \vec{F}_{SURFACE}
\]

\[
\vec{F}_{BODY} = \vec{F}_{SURFACE}
\]

\[
F_D = F_R
\]
TODAY’S PLAN

UNIFORM FLOW & FLOW RESISTANCE

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IV. Reach Resistance
   I. In Uniform Flow
   II. In Natural Channels

V. Mannings Equation

VI. Applying Resistance Equations
If the flow is uniform, there is no acceleration \( \frac{d u}{d x} = 0 \), so \( \sum \vec{F} = m \cdot a \) reduces to \( \sum \vec{F} = 0 \). In other words, there are no net forces acting on the fluid. Thus, the driving forces \( F_d \) are balanced by the resisting forces \( F_r \):

\[
F_D = F_R \quad \text{Eq. 6.6}
\]

How do flow velocity and channel characteristics relate?

- \( \theta \) Slope
- \( W \) Width
- \( Y \) Depth
- \( A \) Cross Sectional Area
- \( P_W \) Wetted Perimeter
- \( U \) Reach Averaged Velocity

French Engineer: Antoine Chézy (1718-1798)
CHEZY PROCEEDED WITH A FORCE BALANCE \( F_D = F_R \)

- He reasoned the downslope component of the gravitational force acting on the water in a reach:

\[
F_D = \gamma \cdot W \cdot Y \cdot X \cdot \sin\theta
\]

or

\[
F_D = \gamma \cdot A \cdot X \cdot \sin\theta
\]

Eq. 6.7

- \( \theta \) Slope
- \( W \) Width
- \( Y \) Depth
- \( A \) Cross Sectional Area
- \( P_W \) Wetted Perimeter
- \( U \) Reach Averaged Velocity

From Dingman (2008) – Chapter 6
SO WHAT ABOUT $F_R$?

- The resistance forces are due to a boundary shear stress $\tau_0$ [FL$^{-2}$] caused by boundary friction:

  \[ \tau_0 = \gamma \cdot Y_W \cdot \sin\theta \]

  From Chapter 5 (eq 5.7)

- But that was at a point, so now applied to the whole cross section. Chézy reasoned that this stress was proportional to the square of the average velocity:

  \[ \tau_0 \propto \gamma \cdot U^2 \]

  \[ \tau_0 = K_T \rho \cdot U^2 \quad \text{Eq. 6.8} \]

  \[ \text{Dimensionless proportionality factor.} \]

Dingman, argues that this reasoning is both dimensionally correct & physically justified based on eq. 3.32, which shows that shear stress $\tau$ (due to turbulence) is proportional to the turbulent velocity fluctuations $u'(t)$, which in turn are ‘proportional to the average velocity’ $U$

\[ \tau_{Tyyx} = -\rho \cdot k \cdot \overline{u_x'(t)} \cdot \overline{u_x'(t)} \quad \text{Eq. 3.32} \]
WRAPPING UP $F_R$

$\tau_0 = K_T \rho \cdot U^2$  \hspace{1cm}  Eq. 6.8

- Since the boundary shear stress $\tau_0$ is a surface force, it acts over the whole area of the channel that is contact with the fluid, $A_B$, which for a rectangular channel is:

$A_B = (2Y + W) \cdot X$  \hspace{1cm}  Eq. 6.9

or

$A_B = P_W \cdot X$

- $\therefore$ applying the boundary shear stress $\tau_0$ over the entire area of the channel in contact with the flow $A_B$ gives us $F_R$:

$F_R = \tau_0 \cdot A_B$  \hspace{1cm}  Eq. 6.10

or

$F_R = K_T \rho \cdot U^2 \cdot P_W \cdot X$
BRINGING TOGETHER $F_D$ & $F_R$

- Recalling $F_D = F_R$, and:
  
  $F_D = \gamma \cdot A \cdot X \cdot \sin \theta$

  $F_R = K_T \rho \cdot U^2 \cdot P_W \cdot X$

- Our force balance is now:
  
  $\gamma \cdot A \cdot X \cdot \sin \theta = K_T \rho \cdot U^2 \cdot P_W \cdot X$

  Eq. 6.11

- Recalling that $\gamma = \rho \cdot g$
  
  $\rho \cdot g \cdot A \cdot X \cdot \sin \theta = K_T \rho \cdot U^2 \cdot P_W \cdot X$

- And solving for $U$:

  
  $\frac{g \cdot A \cdot X \cdot \sin \theta}{K_T \cdot P_W \cdot X} = U^2$

  
  $U = \left( \frac{g}{K_t} \right)^{1/2} \cdot \left( \frac{A}{P_W} \right)^{1/2} \cdot (\sin \theta)^{1/2}$

  Eq. 6.12
CHÉZY’S EQUATION EMERGES...

• Since the **hydraulic radius** $R$ is defined as:

$$R \equiv \frac{A}{P_W} \quad \text{& slope } S \text{ is } \quad S \equiv \sin\theta$$

• We can rewrite

$$U = \left(\frac{g}{K_t}\right)^{1/2} \cdot \left(\frac{A}{P_W}\right)^{1/2} \cdot (\sin\theta)^{1/2}$$

Eq. 6.12

• As

$$U = \left(\frac{1}{K_t}\right)^{1/2} \cdot (g \cdot R \cdot S)^{1/2}$$

Eq. 6.15a

• For very wide channels, $R \approx Y$, so:

$$U = \left(\frac{1}{K_t}\right)^{1/2} \cdot (g \cdot Y \cdot S)^{1/2}$$
More Typical Form

**Box 6.1 Chézy’s C**

In engineering texts, the Chézy equation is usually written as

\[ U = C \cdot (R \cdot S)^{1/2}, \]  

(6B1.1)

where \( C \) expresses the reach conductance and is known as “Chézy’s C.” Note from equation 6.15a that

\[ C \equiv \left( \frac{g}{K_t} \right)^{1/2}, \]  

(6B1.2)

and thus has dimensions \([\text{L}^{1/2} \cdot \text{T}^{-1}]\).

In engineering practice, however, \( C \) is treated as a dimensionless quantity so that it has the same numerical value in all unit systems. This can be a dangerous practice: equation 6B1.1 is in fact correct only if the British (ft-s) unit system is used. If \( C \) is to have the same numerical value in all unit systems, the Chézy equation must be written as

\[ U = u_C \cdot C \cdot (R \cdot S)^{1/2}, \]  

(6B1.3)

where \( u_C \) is a unit-adjustment factor that takes the following values:

<table>
<thead>
<tr>
<th>Unit system</th>
<th>( u_C )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Système Internationale</td>
<td>0.552</td>
</tr>
<tr>
<td>British</td>
<td>1.00</td>
</tr>
<tr>
<td>Centimeter-gram-second</td>
<td>5.52</td>
</tr>
</tbody>
</table>

No systematic method for estimating Chézy’s C from channel characteristics has been published (Yen 2002). The following statistics from a database of 931 flows in New Zealand and the United States collated by the author give a sense of the range of \( C \) values in natural channels:

<table>
<thead>
<tr>
<th>Statistic</th>
<th>( C ) value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>32.5</td>
</tr>
<tr>
<td>Median</td>
<td>29.3</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>17.7</td>
</tr>
<tr>
<td>Maximum</td>
<td>86.6</td>
</tr>
<tr>
<td>Minimum</td>
<td>2.1</td>
</tr>
</tbody>
</table>

From Dingman (2008) – Chapter 6
REVISITING DINGMAN’S CENTRAL PROBLEM

“The central problem of open-channel-flow hydraulics can be stated as follows: Given a channel reach with a specified geometry, material, and slope, what are the relationships among flow depth, average velocity, width and discharge?”

Knowns: $z(x,y), S_0, d_\%$

Unknowns: $Y, U, W, Q$

From Chézy: “The average velocity of a uniform open-channel flow is proportional to the square root of the product of the hydraulic radius $R$ and the downslope component of gravitational acceleration $(g \cdot S)$.”
FOR UNIFORM FLOW... $K_T$ IS THE CHALLENGE

Knobs:
$z(x, y), S_0, d_\%$

$U = \left(\frac{1}{K_t}\right)^{1/2} \cdot (g \cdot R \cdot S)^{1/2}$

Unknowns:
$Y, U, W, Q$

A more complete answer to the central question posed at the beginning of this chapter requires some way of determining the value of $K_T$. This quantity is the proportionality between the shear stress due to the boundary and the square of the velocity; thus, presumably it depends in some way on the nature of the boundary. Most of the rest of this chapter explores the relation between this proportionality and the nature of the boundary. We will see that the velocity profiles derived in chapter 5 along with experimental observations provide much of the basis for formulating this relation. But before proceeding to that exploration, we use the Chézy derivation to formulate the working definition of resistance.

$$K_T = \frac{\tau_0}{\rho \cdot U^2}$$

Eq. 6.8
TODAY’S PLAN

UNIFORM FLOW & FLOW RESISTANCE

I. Boundary Characteristics (for the chapter anyway)
II. Uniform Flow in Open Channels
III. The Chézy Equation
IV. Reach Resistance
   I. In Uniform Flow
   II. In Natural Channels
V. Mannings Equation
VI. Applying Resistance Equations

From Chanson (2004)
**$u_*$ - SHEAR VELOCITY**

- Recall that shear (or friction) velocity $u_* \equiv (g \cdot Y_w \cdot sin\theta_S)^{1/2}$
- $u_*$ can be thought of as a measure of velocity very close to the bed
- We can define a reach-averaged shear velocity
  
  $u_* \equiv (g \cdot R \cdot sin\theta_S)^{1/2}$  
  Eq. 6.16a

- Which for wide channels, can be approximated as:
  
  $u_* \equiv (g \cdot Y \cdot sin\theta_S)^{1/2}$  
  Eq. 6.16b

- Using this definition, we can define a **reach resistance** $\Omega$ as the ratio of shear velocity to reach averaged velocity

  $\Omega \equiv \frac{u_*}{U}$  
  Eq. 6.17f
\[ \Omega \text{ REACH RESISTANCE } \& \ K_T \]

- From \( \Omega \equiv \frac{u_*}{U}, K_T = \frac{\tau_0}{\rho \cdot U^2} \), and \( u_* \equiv (g \cdot Y \cdot \sin \theta_S)^{1/2} \), we can do some algebra, and develop a relationship between \( \Omega \) \& \( K_T \):

\[
\Omega = K_T^{1/2} \quad \text{Eq. 6.18}
\]

- We can substitute \( \Omega \) back into Chézy

\[
U = \left( \frac{1}{K_t} \right)^{1/2} \cdot (g \cdot R \cdot S)^{1/2}
\]

\[
U = \frac{1}{\Omega} \cdot (g \cdot R \cdot S)^{1/2}
\]

\[
U = \Omega^{-1} \cdot u_* \quad \text{Eq. 6.19}
\]
AS A SIDE NOTE….  

DARCY-WEISBACH FRICTION FACTOR

**BOX 6.2** The Darcy-Weisbach Friction Factor

In 1845 Julius Weisbach (1806–1871) published the results of pioneering experiments to determine frictional resistance in pipe flow (Rouse and Ince 1963) and formulated a dimensionless factor, $f_{DW}$, that expresses this resistance:

$$f_{DW} = 2 \left( \frac{h_e}{X} \right) \left( \frac{D \cdot g}{U^2} \right). \quad (682.1)$$

where $h_e$ (L) is the loss in mechanical energy per unit weight of water, or head (see equation 4.45) in distance $X$, $D$ is the pipe diameter, $U$ is the average flow velocity, and $g$ is gravitational acceleration. In 1857, the same Henry Darcy (1803–1858) whose experiments led to Darcy's law, the central formula of groundwater hydraulics, published the results of similar pipe experiments, and $f_{DW}$ is known as the Darcy-Weisbach friction factor.

The pipe diameter $D$ equals four times the hydraulic radius, $R$, so

$$f_{DW} = 8 \left( \frac{h_e}{X} \right) \left( \frac{R \cdot g}{U^2} \right). \quad (682.2)$$

The quantity $h_e/X$ in pipe flow is physically identical to the channel and water-surface slope, $S = \sin \theta$, in uniform open-channel flow, so the friction factor for open-channel flow is

$$f_{DW} = 8 \cdot \frac{g \cdot R \cdot S}{U^2}. \quad (682.3a)$$

From the definition of shear velocity, $u_s$ (equation 6.16a), 682.3a can also be written as

$$f_{DW} = 8 \cdot \frac{u_s^2}{U^2}. \quad (682.3b)$$

and from the definition of $\Omega$ (equation 6.17), we see that

$$f_{DW} = 8 \cdot \Omega^2. \quad (682.4a)$$

$$\Omega = \left( \frac{f_{DW}}{8} \right)^{1/2} = 0.354 \cdot f_{DW}^{1/2}. \quad (682.4b)$$

The Darcy-Weisbach friction factor is commonly used to express resistance in open channels as well as pipes. However, the $\Omega$ notation is used herein because it is simpler: It does not include the 8 multiplier and is written in terms of $u_s$ and $U$ rather than the squares of those quantities.
TODAY’S PLAN

UNIFORM FLOW & FLOW RESISTANCE

I. Boundary Characteristics (for the chapter anyway)
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   I. In Uniform Flow
   II. In Natural Channels
V. Mannings Equation
VI. Applying Resistance Equations

From Chanson (2004)
RELATIVE ROUGHNESS & SMOOTHNESS

- What is the difference between relative roughness and relative smoothness?
- \( \frac{y_r}{Y} \) or \( \frac{Y}{y_r} \)
- Clean relationship to resistance \( \Omega \)
- Why is relationship non-linear?

Figure 6.9 Baseline resistance, \( \Omega_* \), as a function of relative smoothness, \( Y/Y_* \), for fully rough turbulent flow in wide channels as given by equation 6.25. This is identical to the friction given by the integrated F-VK velocity profile (equation 6.26). (a) Arithmetic plot; (b) semi-logarithmic plot.
MOODY DIAGRAM

- See pp. 223-224
- What is eq. 6.20 saying?
- Reynolds Number vs. Resistance...

Figure 6.8 The Moody diagram: Relation between resistance, $\Omega$; Reynolds number, $Re$; and relative smoothness, $Y/y_r$, for laminar, smooth turbulent, and rough turbulent flows in wide open channels. $Y/y_r$ affects resistance only for rough turbulent flows ($Re > 2000$ and $Re_b > 5$). The effect of $Re$ on resistance in rough turbulent flows decreases with $Re$; resistance becomes independent of $Re$ for “fully rough” flows ($Re_b > 70$).
TODAY’S PLAN

UNIFORM FLOW & FLOW RESISTANCE

I. Boundary Characteristics (for the chapter anyway)
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From Chanson (2004)
CHANNEL IRREGULARITIES...

- Let’s discuss this section...

Figure 6.11 Three categories of channel irregularity that cause changes in the magnitude and/or direction of velocity vectors and hence increase flow resistance beyond that given by equation 6.25. (a) Irregularities in cross-section. (b) Irregularities in plan (map) view. ζ designates sinuosity, the streamwise distance \( \Delta X \) divided by the valley distance \( \Delta X_v \); \( r_c \) is the radius of curvature of a river bend, \( \lambda_m \) is meander wavelength, \( a_m \) is meander amplitude, and \( a_c \) represents the centrifugal acceleration. (c) Reach-scale irregularities in longitudinal profile (channel slope); these are more pronounced at low flows and less pronounced at high flows.
TERRESTRIAL LASER SCANNING

CHARACTERISTICS

- New generation of tripod-mounted laser scanners
- 6-12 sec angular resolution (>4000 points/second)
- 4-40 KHz Sampling rate
- 250-350 m range
- Generate 3D x,y,z point cloud data

OPPORTUNITIES

- High sampling rate
  - Rapid survey acquisition times
- Dense spatial sampling
  - Previously unachievable sampling density over wide regions
- Precise individual measurements
  - ~total station quality
- Non-invasive
  - Observation and recording without interference
- Fully 3D datasets
  - cf single perspective airborne lidar
ALTERNATIVE MEASURE OF $y_r$
> 250,000,000 xyz
Photorendered Point cloud

12 scans registered

Density (median) = \( \sim 1400 \text{pts/m}^2 \)

> 250 coincident tie-points

RMSE:
Control = 7 mm
Check = 8 mm
ROUGHNESS & VERIFICATION

- Reach Scale Terrestrial Laser Scanning
- Select homogenous (GSD) patches
- Total Station profiling over the patches
- Pebble count in the selected patches
DETRENDING THE $\sigma$ OF THE TLS POINT CLOUD

Filtering tool box $\rightarrow n, Z_{\text{min}}, Z_{\text{max}}, Z_{\text{mean}}, \sigma, \sigma_d$
(per each centre point)
Do I really care about grain sizes?
TODAY’S PLAN

UNIFORM FLOW & FLOW RESISTANCE

I. Boundary Characteristics (for the chapter anyway)
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   I. In Uniform Flow
   II. In Natural Channels
V. Mannings Equation
VI. Applying Resistance Equations
MANNING’S EQUATION

- Forget the theoretical derivation...
- He reviewed lots of field & lab research based on Chézy equation & used an empirical formula that best fit the experimental results:

\[ U = K_m \cdot R^{2/3} \cdot S_s^{1/2} \]

- Compared with Chézy:

\[ U = \frac{1}{\Omega} \cdot (g \cdot R \cdot S)^{1/2} \]
RECALL..

BOX 6.2 The Darcy-Weisbach Friction Factor

In 1845 Julius Weisbach (1806–1871) published the results of pioneering experiments to determine frictional resistance in pipe flow (Rouse and Ince 1963) and formulated a dimensionless factor, \( f_{\text{DW}} \), that expresses this resistance:

\[
f_{\text{DW}} = 2 \left( \frac{h_c}{X} \right) \left( \frac{D g}{U^2} \right).
\]

(682.1)

where \( h_c \) (L) is the loss in mechanical energy per unit weight of water, or head (see equation 4.45) in distance \( X \), \( D \) is the pipe diameter, \( U \) is the average flow velocity, and \( g \) is gravitational acceleration. In 1857, the same Henry Darcy (1803–1858) whose experiments led to Darcy’s law, the central formula of groundwater hydraulics, published the results of similar pipe experiments, and \( f_{\text{DW}} \) is known as the **Darcy-Weisbach friction factor**.

The pipe diameter \( D \) equals four times the hydraulic radius, \( R \), so

\[
f_{\text{DW}} = 8 \left( \frac{h_c}{X} \right) \left( \frac{R g}{U^2} \right).
\]

(682.2)

The quantity \( h_c/X \) in pipe flow is physically identical to the channel and water-surface slope, \( S = \sin \theta \), in uniform open-channel flow, so the friction factor for open-channel flow is

\[
f_{\text{DW}} = 8 \left( \frac{g R S}{U^2} \right).
\]

(682.3a)

From the definition of shear velocity, \( u_* \) (equation 6.16a), 682.3a can also be written as

\[
f_{\text{DW}} = 8 \left( \frac{u_*^2}{U^2} \right).
\]

(682.3b)

and from the definition of \( \Omega \) (equation 6.17), we see that

\[
f_{\text{DW}} = 8 \Omega^2
\]

(682.4a)

\[
\Omega = \left( \frac{f_{\text{DW}}}{8} \right)^{1/2} = 0.354 f_{\text{DW}}^{1/2}
\]

(682.4b)

The Darcy-Weisbach friction factor is commonly used to express resistance in open channels as well as pipes. However, the \( \Omega \) notation is used herein because it is simpler: It does not include the 8 multiplier and is written in terms of \( u_* \) and \( U \) rather than the squares of those quantities.

Unidirectional water flow

that is discussed later. Combining Equations (5.22) and (5.37) results in a relationship involving depth-averaged flow velocity, depth \( d \), slope, and bed roughness in steady, uniform flows:

\[
U = \sqrt{8gdS/f}
\]

(5.38)

This is called the d’Arcy–Weisbach (or Darcy–Weisbach) equation. The Chezy equation is similar and given by

\[
U = C \sqrt{dS}
\]

(5.39)

where \( C = \sqrt{8gd/f} \). A variation on these equations is the Manning equation:

\[
U = 1.49d^{2/3} S^{1/2}/n
\]

(5.40)

The Chezy and Manning equations are inferior to the Darcy–Weisbach equation because their resistance coefficients (\( C \) and \( n \), respectively) are dimensional, as opposed to \( f \), which is dimensionless. Thus the American Society of Civil Engineers recommended discontinuing use of the Manning equation in 1966. However, it is still in use by many engineers worldwide.
THE PROBLEM WITH MANNINGS

\[ U = K_M \cdot R^{2/3} \cdot S_S^{1/2}, \]  
\[ (6.40a) \]

where \( K_M \) is a proportionality constant representing reach conductance. For historical reasons (see Dooge 1992), subsequent researchers replaced \( K_M \) by its inverse, \( 1/n_M \), and wrote the equation as

\[ U = \left( \frac{1}{n_M} \right) \cdot R^{2/3} \cdot S_S^{1/2}, \]  
\[ (6.40b) \]

called Manning’s equation, where the resistance factor \( n_M \) is called Manning’s \( n \).

Manning’s equation has come to be accepted as “the” resistance equation for open-channel flow, largely replacing the Chézy equation in practical applications. The essential difference between the two is that the hydraulic-radius exponent is \( 2/3 \) rather than \( 1/2 \). This difference is important because it makes the Manning equation dimensionally inhomogeneous. As with Chézy’s \( C \) (see box 6.1), values of \( n_M \) are treated as constants for all unit systems, and in order to give correct results, the Manning equation must be written as

\[ U = u_M \cdot \left( \frac{1}{n_M} \right) \cdot R^{2/3} \cdot S_S^{1/2}, \]  
\[ (6.40c) \]

where \( u_M \) is a unit-adjustment factor that takes the following values:

<table>
<thead>
<tr>
<th>Unit system</th>
<th>( u_M )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Système Internationale</td>
<td>1.00</td>
</tr>
<tr>
<td>British</td>
<td>1.49</td>
</tr>
<tr>
<td>Centimeter-gram-second</td>
<td>4.64</td>
</tr>
</tbody>
</table>
**Table 6.3** General approaches to a priori estimation of Manning’s $n_M$.

<table>
<thead>
<tr>
<th>Approach</th>
<th>Comments</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Visual comparison with photographs of channels for which $n_M$ has been measured (see table 6.4)</td>
<td>Expedient method; subjective, dependent on operator experience; subject to considerable uncertainty</td>
<td>Faskin (1963), Barnes (1967), Arcement et al. (1989), Hicks and Mason (1991)</td>
</tr>
<tr>
<td>2. Tables of typical $n_M$ values for reaches of various materials and types (see table 6.5)</td>
<td>Expedient method; subjective, dependent on operator experience; subject to considerable uncertainty</td>
<td>Chow (1959), French (1985)</td>
</tr>
<tr>
<td>3. Formulas that account for components of reach resistance (see table 6.6)</td>
<td>Expedient method; more objective than approaches 1 and 2 but lacks theoretical basis</td>
<td>Cowan (1956), Faskin (1963), Arcement et al. (1989)</td>
</tr>
<tr>
<td>4. Formulas that relate $n_M$ to bed-sediment grain size $d_p$ (see table 6.7)</td>
<td>Require measurement of bed sediment; reliable only for straight quasi-prismatic channels where bed roughness is the dominant factor contributing to resistance</td>
<td>Chang (1988), Marus et al. (1992)</td>
</tr>
<tr>
<td>5. Formulas that relate $n_M$ to hydraulic radius and relative smoothness</td>
<td>Require measurement of bed sediment, depth, and slope; forms are based on theory; coefficients are based on field measurement; can give good results in conditions similar to those for which established</td>
<td>Limerinos (1970), Bathurst (1985)</td>
</tr>
<tr>
<td>6. Statistical formulas that relate $n_M$ to measurable flow parameters (see table 6.8)</td>
<td>Can provide good estimates, especially useful when bed-material information is lacking, as in remote sensing, but subject to considerable uncertainty</td>
<td>Riggs (1976), James (1984), Dingman et al. (1997), Sharma (1997), Bjerklie et al. (2000)</td>
</tr>
</tbody>
</table>

**Table 6.4** Summary of reports presenting photographs of reaches for which Manning’s $n_M$ has been measured.

<table>
<thead>
<tr>
<th>Types of reach</th>
<th>No. of reaches</th>
<th>No. of flows</th>
<th>Minimum $n_M$</th>
<th>Maximum $n_M$</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canals and dredged channels (USA)</td>
<td>48</td>
<td>326</td>
<td>0.014</td>
<td>0.162</td>
<td>Faskin (1963)</td>
</tr>
<tr>
<td>Natural rivers (USA)</td>
<td>51</td>
<td>62</td>
<td>0.024</td>
<td>0.075</td>
<td>Barnes (1967)</td>
</tr>
<tr>
<td>Flood plains (USA)</td>
<td>16</td>
<td>16</td>
<td>__a</td>
<td>__a</td>
<td>Arcement et al. (1989)</td>
</tr>
<tr>
<td>Natural rivers (New Zealand)</td>
<td>78</td>
<td>559</td>
<td>0.016</td>
<td>0.270</td>
<td>Hicks and Mason (1991)</td>
</tr>
</tbody>
</table>

__a__ See reference for methodology for computing composite (channel plus flood plain) $n_M$ values.
Table 6.5 Values of Manning’s $n_M$ for natural streams.

<table>
<thead>
<tr>
<th>Channel description</th>
<th>Minimum</th>
<th>Normal</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minor streams (bankfull width &lt; 100 ft)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Streams on plain</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Clean, straight, full stage, no riffles or deep pools</td>
<td>0.025</td>
<td>0.030</td>
<td>0.033</td>
</tr>
<tr>
<td>2. Same as above, but more stones and weeds</td>
<td>0.030</td>
<td>0.035</td>
<td>0.040</td>
</tr>
<tr>
<td>3. Clean, winding, some pools and shoals</td>
<td>0.033</td>
<td>0.040</td>
<td>0.045</td>
</tr>
<tr>
<td>4. Same as above, but some weeds and stones</td>
<td>0.035</td>
<td>0.045</td>
<td>0.050</td>
</tr>
<tr>
<td>5. Same as above, but lower stages, more ineffective slopes and sections</td>
<td>0.040</td>
<td>0.048</td>
<td>0.055</td>
</tr>
<tr>
<td>6. Same as item 4, but more stones</td>
<td>0.045</td>
<td>0.050</td>
<td>0.060</td>
</tr>
<tr>
<td>7. Sluggish reaches, weedy, deep pools</td>
<td>0.050</td>
<td>0.070</td>
<td>0.080</td>
</tr>
<tr>
<td>8. Very weedy reaches, deep pools, or floodways with heavy stand of timber and underbrush</td>
<td>0.075</td>
<td>0.100</td>
<td>0.150</td>
</tr>
<tr>
<td>Mountain Streams</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No vegetation in channel, banks usually steep, trees and brush along banks submerged at high stages</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Bottom: gravels, cobbles, and few boulders</td>
<td>0.030</td>
<td>0.040</td>
<td>0.050</td>
</tr>
<tr>
<td>2. Bottom: cobbles with large boulders</td>
<td>0.040</td>
<td>0.050</td>
<td>0.070</td>
</tr>
<tr>
<td>Major Streams (bankfull width &gt; 100 ft)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Regular section with no boulders or brush</td>
<td>0.025</td>
<td>—</td>
<td>0.060</td>
</tr>
<tr>
<td>2. Irregular and rough section</td>
<td>0.035</td>
<td>—</td>
<td>0.100</td>
</tr>
<tr>
<td>Floodplains</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Short grass, no brush</td>
<td>0.025</td>
<td>0.030</td>
<td>0.035</td>
</tr>
<tr>
<td>2. High grass, no brush</td>
<td>0.030</td>
<td>0.035</td>
<td>0.050</td>
</tr>
<tr>
<td>3. Cultivated area, no crop</td>
<td>0.020</td>
<td>0.030</td>
<td>0.040</td>
</tr>
<tr>
<td>4. Mature row crops</td>
<td>0.025</td>
<td>0.035</td>
<td>0.045</td>
</tr>
<tr>
<td>5. Mature field crops</td>
<td>0.030</td>
<td>0.040</td>
<td>0.050</td>
</tr>
<tr>
<td>6. Scattered brush, heavy weeds</td>
<td>0.035</td>
<td>0.050</td>
<td>0.070</td>
</tr>
<tr>
<td>7. Light brush and trees, in winter</td>
<td>0.035</td>
<td>0.050</td>
<td>0.060</td>
</tr>
<tr>
<td>8. Light brush and trees, in summer</td>
<td>0.040</td>
<td>0.060</td>
<td>0.080</td>
</tr>
<tr>
<td>9. Medium to dense brush, in winter</td>
<td>0.045</td>
<td>0.070</td>
<td>0.110</td>
</tr>
<tr>
<td>10. Medium to dense brush, in summer</td>
<td>0.070</td>
<td>0.100</td>
<td>0.140</td>
</tr>
<tr>
<td>11. Dense willows, summer, straight</td>
<td>0.110</td>
<td>0.150</td>
<td>0.200</td>
</tr>
<tr>
<td>12. Cleared land with tree stumps, no sprouts</td>
<td>0.030</td>
<td>0.040</td>
<td>0.050</td>
</tr>
<tr>
<td>13. Same as above, but with heavy growth of sprouts</td>
<td>0.050</td>
<td>0.060</td>
<td>0.080</td>
</tr>
<tr>
<td>14. Heavy stand of timber, a few down trees, little undergrowth, flood stage below branches</td>
<td>0.080</td>
<td>0.100</td>
<td>0.120</td>
</tr>
<tr>
<td>15. Same as above, but with flood stage reaching branches</td>
<td>0.100</td>
<td>0.120</td>
<td>0.160</td>
</tr>
</tbody>
</table>

From Chow (1959, table 5.6). Reproduced with permission of McGraw-Hill.
### RELATING GRAIN SIZE TO $n$

Table 6.7 Formulas relating Manning’s $n_M$ to bed-sediment size and relative smoothness (grain diameters $d_p$, in mm; hydraulic radius, $R$, in m).

<table>
<thead>
<tr>
<th>Formula</th>
<th>Remarks</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_M$ or $n_0 = 0.015 \cdot d^{1/6}$</td>
<td>Original “Strickler formula” for uniform sand</td>
<td>Strickler (1923) as reported by Chang (1988)</td>
</tr>
<tr>
<td>$n_M$ or $n_0 = 0.0079 \cdot d^{1/6}$</td>
<td></td>
<td>Keulegan (1938) as reported by Marcus et al. (1992)</td>
</tr>
<tr>
<td>$n_M$ or $n_0 = 0.0122 \cdot d^{1/6}$</td>
<td>Sand mixtures</td>
<td>Meyer-Peter and Muller (1948)</td>
</tr>
<tr>
<td>$n_M$ or $n_0 = 0.015 \cdot d^{1/6}$</td>
<td>Gravel lined canals</td>
<td>Lane and Carlson (1938) as reported by Chang (1988)</td>
</tr>
<tr>
<td>$n_M$ or $n_0 = \frac{R^{1/6}}{[7.69 \cdot \ln(R/d_{84}) + 63.4]}$</td>
<td>Gravel streams with slope $&gt; 0.004$</td>
<td>Limerinos (1970)</td>
</tr>
<tr>
<td>$n_M$ or $n_0 = \frac{R^{1/6}}{[7.64 \cdot \ln(R/d_{84}) + 65.3]}$</td>
<td></td>
<td>Bathurst (1985)</td>
</tr>
<tr>
<td>$n_M$ or $n_0 = \frac{R^{1/6}}{[7.83 \cdot \ln(R/d_{84}) + 72.9]}$</td>
<td>Derived from P-vK law for wide channels</td>
<td>Dingman (1984)</td>
</tr>
</tbody>
</table>
TODAY’S PLAN

UNIFORM FLOW & FLOW RESISTANCE

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VI. Applying Resistance Equations
HOW TO GET Q vs. Y WHERE YOU DON’T HAVE IT

**BOX 6.6 Steps for Estimating Velocity–Discharge and Depth–Discharge Relations for an Ungaged Reach**

1. Using the techniques of box 2.1, identify the bankfull elevation through the reach.
2. Using the techniques of box 2.2 [1. Channel (Bankfull) Geometry], survey a typical cross section to determine the channel geometry.
3. Determine the size distribution of bed sediment, $d_p$. [See section 2.3.2.1. Refer to Bunte and Abt (2001) for detailed field procedures.]
4. Survey water-surface elevation through the reach to determine water-surface slope, $S_s$. [Refer to Harrelson et al. (1994) for detailed survey procedures.]
5. Select a range of elevations up to bankfull.
6. Using the techniques of box 2.2 (2. Geometry at a Subbankfull Flow), determine water-surface width $W$, cross-sectional area $A$, and average depth $Y = A/W$ associated with each selected elevation.
7. Estimate reach resistance: (a) If using the Chézy equation, use results of steps 3–6 to estimate $\Omega_a$ via equation 6.25 for each selected elevation and adjust to give $\Omega$ based on considerations of section 6.6. (b) If using the Manning equation, use one of the methods of section 6.8.2 to estimate Manning’s $n_M$.
8. Assume hydraulic radius $R = Y$ and estimate average velocity $U$ for each selected elevation via either the Chézy equation (equation 6.15a) or the Manning equation (equation 6.40).
9. Estimate discharge as $Q = U \cdot A$ for each selected elevation.
10. Use results to generate plots of $U$ versus $Q$ and $Y$ versus $Q$.

*Figure 6.25* Surveyed cross section in the center of the Hutt River reach shown in figure 6.24. Elevations are relative to the lowest elevation in the cross section. The dashed lines are the actual river levels at the maximum depths ($\Psi$) indicated; $\Psi_B$ is the bankfull maximum depth. Note approximately 10-fold vertical exaggeration.
**BOX 6.7** Example Computation of Channel Geometry: Hutt River at Kapite, New Zealand

The line of a cross section is oriented at right angles to the general flow direction. An arbitrary zero point is established at one end of the line; by convention, this is usually on the left bank (facing downstream), but it can be on either bank. Points are selected along the line to define the cross-section shape; these are typically “slope breaks”—points where the ground-surface slope changes. An arbitrary elevation datum is established, and the elevations of these points above this datum are determined by surveying (see Harreton et al. 1994). To illustrate the computations, we use data for cross section of the Hutt River in New Zealand (figure 6.24). Section survey results are recorded as elevations, $z_i$, at distances along the section line, $w_i$. At each point, the local bankfull depth $y_{BF_i}$ can be calculated as

$$y_{BF_i} = \psi_{BF_i} - z_i,$$  \hspace{1cm} (687.1)

where $\psi_{BF_i}$ is the bankfull maximum depth. The data for the Hutt River section are given in table 687.1 and are plotted in figure 6.25.

**Table 687.1**

<table>
<thead>
<tr>
<th>$w_i$ (m)</th>
<th>0.0</th>
<th>1.0</th>
<th>5.5</th>
<th>7.5</th>
<th>9.0</th>
<th>10.0</th>
<th>11.2</th>
<th>13.3</th>
<th>13.4</th>
<th>14.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z_i$ (m)</td>
<td>3.78</td>
<td>3.71</td>
<td>2.72</td>
<td>2.18</td>
<td>1.92</td>
<td>1.50</td>
<td>0.96</td>
<td>0.86</td>
<td>0.85</td>
<td>0.84</td>
</tr>
<tr>
<td>$\psi_{BF_i}$ (m)</td>
<td>0.00</td>
<td>0.07</td>
<td>1.06</td>
<td>1.60</td>
<td>1.86</td>
<td>2.28</td>
<td>2.82</td>
<td>2.92</td>
<td>3.13</td>
<td>3.24</td>
</tr>
<tr>
<td>$w_i$ (m)</td>
<td>17.5</td>
<td>19.8</td>
<td>19.9</td>
<td>20.6</td>
<td>21.3</td>
<td>24.0</td>
<td>25.8</td>
<td>27.7</td>
<td>28.8</td>
<td>30.0</td>
</tr>
<tr>
<td>$z_i$ (m)</td>
<td>0.53</td>
<td>0.58</td>
<td>0.32</td>
<td>0.28</td>
<td>0.41</td>
<td>0.30</td>
<td>0.44</td>
<td>0.12</td>
<td>0.00</td>
<td>0.24</td>
</tr>
<tr>
<td>$\psi_{BF_i}$ (m)</td>
<td>3.25</td>
<td>3.20</td>
<td>3.46</td>
<td>3.50</td>
<td>3.37</td>
<td>3.49</td>
<td>3.34</td>
<td>3.66</td>
<td>3.78</td>
<td>3.54</td>
</tr>
</tbody>
</table>

Once the section is plotted, several arbitrary elevations are identified to represent water-surface elevations (the horizontal lines in figure 6.25). For each level, the horizontal positions of the left- and right-hand intersections of the level line with the channel bottom are determined and identified as $w_i$ and $w_R$, respectively. For each selected elevation, the water-surface width $W$ is

$$W = |w_R - w_i|.$$  \hspace{1cm} (687.2)

Selecting the level $\psi = 2$ m in the Hutt River cross section for example calculations, we see from figure 6.25 that

$$W = |41.5 - 8.5| = 33.0 \text{ m}.$$

(Continued)
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From Chanson (2004)