Watershed Sciences 4930 & 6920
GEOGRAPHIC INFORMATION SYSTEMS

WEEK EIGHT– Lecture
INTERPOLATION & SPATIAL ESTIMATION

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HOUSEKEEPING

• Quizzes
• Lab 8?

READING FOR TODAY

TODAY’S PLAN

I. Sampling
II. Spatial Interpolation
   I. TIN
   II. Nearest Neighbor
   III. Fixed Radius - Local Averaging
   IV. Inverse Distance Weighted
   V. Natural Neighbor
   VI. Spline
   VII. TOPO
   VIII. Kriging
III. Interpolations Compared

WHAT CAN WE COLLECT AT POINTS?

• Think of everything we can....

DIFFERENT SAMPLING STRATEGIES

• A) Regular Grid (systematic)
• B) Random
• C) Cluster
• D) Adaptive
**SAMPLING PATTERNS WE’VE DISCUSSED**

- Point Based Sampling
- Sampling Patterns?
  - Uniform grid?
  - Sections?
  - Irregular
  - Random
  - Feature-based

**A FEW OTHER VARIANTS...**

Sampling Pattern: appropriate arrangement of samples considering both spatial interpolation and statistical inference

**WHAT’S WRONG WITH OUR SAMPLING?**

- NOT spatially continuous over area of interest
- Can’t afford (time and money) to have samples everywhere
- Even if finite number of objects in study area, usually too many to measure all...
- Some sites impossible or unfeasible to visit

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**INTERPOLATION DEFINED**

‘Interpolation predicts values for cells in a raster from a limited number of sample data points. It can be used to predict unknown values for any geographic point data, such as elevation, rainfall, chemical concentrations, noise levels, and so on.’

**NATIONAL AIR QUALITY...**

Interpolated in space from 50ish stations...
Interpolated in time through temporal averaging

From Bolstad (2008) Chapter 12
WHY INTERPOLATE?


Easier to see spatial patterns and trends in surfaces than raw discrete data.

WHY INTERPOLATE?


Because variations from the mean might matter.

THE AVERAGE IS HARDLY ANYWHERE!


DIFFERENT INTERPOLATIONS ALSO GIVE DIFFERENT RESULTS....

These differences are less problematic.... But worth considering....

INTERPOLATION TYPES VARY IN SOPHISTICATION


WHAT DO WE USE INTERPOLATION FOR?

- Interpolation from point observations to:
  - A continuous surface, which can be:
    - Vector Polygons (e.g. TIN or Theisan Polygons)
    - Raster Grid
  - Interpolation from Vector Polygons (e.g. TIN or Theisan Polygons) to:
    - Raster Grid
  - Extraction (by interpolation) from raster to points

Because we want to know approximate values at areas where we don't have observations.
IS YOUR RASTER A GRID OR LATTICE?

• Is this just a display thing... or is it algorithmic?
• Grid centered or node centered?

From Berry (2009): GIS Modeling & Analysis
http://www.innovativegis.com

POINT EXTRACTION

• Points to be extracted won’t line up exactly on a cell center... so what is the value?
• If ‘interpolation’ checked, it uses a bilinear interpolation?

From Bolstad (2008) Chapter 12

SOME INTERPOLATION EXAMPLES

Interpolating a rainfall surface

The data here is a point dataset of mean rainfall levels, shown by the statistics on the left. The illustration on the right shows a raster interpolated surface. The surface values are produced using 'inverse distance weighted' interpolation, using rainfall values of nearby stations.

Interpolating a concentration surface

In this example, the interpolation tool was used to study the correlation of the mean concentrations of key diseases in California. The image on the left shows the point data points. The image on the right shows the interpolated surface, showing prominent areas of high disease concentration.

A DATASET WE’LL KEEP LOOKING AT...

Elevation points.... DEM

From Bolstad (2008) Chapter 12

ANOTHER DATASET WE’LL KEEP LOOKING AT

TIN & CONTOUR EXERCISE

TIN & CONTOUR EXERCISE

INTERPOLATION IN ARCGIS

• Three primary places:
  1. 3D Analyst Tools
  2. Geostatistical Analyst Tools
  3. Spatial Analyst Tools
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BUILDING A TIN FROM VECTOR DATA

A Triangular Irregular Network (TIN) is the simplest and most common interpolation technique for building surfaces with irregularly spaced elevation data (McCullagh, 1981).

TIN to RASTER

- Natural neighbors vs. Linear
- An interpolation choice when going from vector to raster...

RESAMPLING (A FORM OF INTERPOLATION)

1. Nearest Neighbor Assignment
2. Bilinear Interpolation
3. Cubic Convolution

• Good for discrete (categorical) data since it does not alter value of input
• Once location of cell’s center on output raster located on input raster, nearest neighbor assignment determines location of the closest cell center

RESAMPLING (A FORM OF INTERPOLATION)

1. Nearest Neighbor Assignment
2. Bilinear Interpolation
3. Cubic Convolution

• Uses values of 4 nearest input cell centers to determine the value of output
• New value is a distance-weighted average
• Results in smoother-looking surface then nearest neighbor...

RESAMPLING (A FORM OF INTERPOLATION)

1. Nearest Neighbor Assignment
2. Bilinear Interpolation
3. Cubic Convolution

• Similar to bilinear interpolation except that weighted average is calculated from the 16 nearest input cell centers and their values
**RECALL OUR TIN -> DEM**

**TIN & CONTOUR EXERCISE**

**RESAMPLE EXAMPLE**

- Resampling choices...
- 10 m to 30 m Raster

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8. Kriging
9. Interpolations Compared

**THIESEN (VORONI) POLYGONS**

- First build the olive polygons by bisecting lines of perimeter polygon
- Then build the central peach polygon by bisecting lines to each of surrounding points and connecting midpoints
- Every polygon has ONLY one point to be interpolated!

This is AKA Nearest Neighbor... (not to be confused with Natural Neighbors)

**IN ARC...**

- Use Create Thiessen Polygons
- Notice
  - Influence of point spacing
  - Extension outside of sampled area

**NEAREST NEIGHBOR (AKA NATURAL NEIGHBOR)**

The points are not interpolated in this example, the region (Thiessen polygon) is...

From Bolstad (2008) Chapter 22
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Fixed Radius Interpolation

- From Bolstad (2008)
- Chapter 12

Fixed Radius Applied

- Not an exact interpolator… Why?

Inverse Distance Weighted

- Estimates a value for a location as the weighted-average of the nearby data values within the roving window
- Average weighted so influence of surrounding values decrease with increasing distance from location being estimated

\[
\hat{z}_j = \frac{\sum_{i=1}^{n} \frac{z_i}{d_{ij}^2}}{\sum_{i=1}^{n} \frac{1}{d_{ij}^2}}
\]
INVERSE DISTANCE WEIGHTED

- Value of unknown points estimated using sampled values and distance to those samples
- The weight (or importance) of each sample point is determined by its distance away

\[ z = \frac{\sum_{i=1}^{n} \frac{z_i}{d_i^2}}{\sum_{i=1}^{n} \frac{1}{d_i^2}} \]

From Bolstad (2008) Chapter 12

IDW EXAMPLE...

IDW SENSITIVITY

- Exponent \( n \) as a point’s influence deteriorates with distance
- Number of points controls search distance
- Local influences are stronger as both \( n \) and \( i \) increase

From Bolstad (2008) Chapter 12

INVERSE DISTANCE WEIGHTED

- Method assumes variable being mapped decreases in influence with distance from sampled location
- Notice interpolation extent

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NATURAL NEIGHBOR

• Not to be confused with nearest neighbor...

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SPLINE INTERPOLATION 1D

• A flexible ruler...
• Can be represented with series of polynomial equations
• Same principle as line functions (right) applied to surface splines...
• Equations can be first order (linear), 2nd order (quadratic) and higher order polynomial fits

SPLINE INTERPOLATION 2D

• Designed to minimize surface curvature
• Creates smooth surface that passes exactly through input points

SPLINE
WHAT IS KRIGING?

- An advanced geostatistical procedure that generates an estimated surface from a scattered set of points with z-values.
- Unlike other interpolation methods, Kriging involves an interactive investigation of the spatial behavior of the phenomenon represented by the z-values before you select the best estimation method for generating the output surface.

The kriging formula:

\[ z(\mathbf{x}) = \sum_{i=1}^{n} w_i z_i(\mathbf{x}) \]

- \( z(\mathbf{x}) \) is the measured value at the ith location.
- \( w_i \) is the weight for the measured value at the ith location.
- \( z_i(\mathbf{x}) \) is the prediction for the measured value at the ith location.
- \( n \) is the number of measured values.

TOPO TO RASTER

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TOPO TO RASTER

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TWO STEPS....

- Statistically based estimator of spatial variables
- Relies on:
  - Spatial Trend – increase or decrease in a variable in some direction
  - Local spatial autocorrelation
  - Random stochastic variation
THE FIRST STEP

FIT EMPIRICAL DATA TO SEMIVARIOGRAM

• **Models:**
  - Circular
  - Spherical
  - Exponential
  - Gaussian
  - Linear

STEP 2: MAKING THE PREDICTION

• Now you’ve uncovered dependence (autocorrelation) in your data, we can make a prediction using fitted model (set aside empirical semivariogram)
  
• Like IDW, kriging forms weights from surrounding measured values to predict unmeasured locations... Instead of just using distance, kriging uses semivariogram
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INTERPOLATION METHODS COMPARED

<table>
<thead>
<tr>
<th>Method</th>
<th>Speed</th>
<th>Type</th>
<th>Comments</th>
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<tbody>
<tr>
<td>TIN with Linear Interpolation</td>
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<tr>
<td>Local Polynomial</td>
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| Interpolation Techniques Compared

CONCISE ARTICLE SUMMARY

Barriers, which reflect the presence of faults, shear, or topographic features that reduce linear connectivity in a surface, also control how surfaces are generated. DEM and Kriging support the use of barriers.
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